Integrales inmediatas de tipo potencial

- $\int x^2(3x^3+14)^3\,dx = \frac{1}{9}\int 9x^2(3x^3+14)^3\,dx = \frac{1}{9}\frac{1}{4}\int 4\cdot 9x^2(3x^3+14)^3\,dx =$

  $= \frac{1}{36}(3x^3+14)^4 + C$

- $\int \sqrt[5]{5x+6}\,dx = \int (5x+6)^{1/5}\,dx = \frac{1}{5}\int 5(5x+6)^{1/5}\,dx =$

  $= \frac{1}{5}\frac{(5x+6)^{1/5+1}}{\frac{1}{5}+1} = \frac{1}{6}\sqrt[5]{(5x+6)^6} + C$

- $\int \frac{17x}{\sqrt[3]{6x^2+8}}\,dx = 17\int x(6x^2+8)^{-1/3}\,dx = 17\frac{1}{12}\int 12x(6x^2+8)^{-1/3}\,dx =$

  $= \frac{17}{12}\frac{(6x^2+8)^{\frac{-1}{3}+1}}{\frac{-1}{3}+1} = \frac{51}{24}\sqrt[3]{(6x^2+8)^2} + C$

- $\int \frac{\operatorname{arctg} x}{1+x^2}\,dx = \int \frac{1}{1+x^2}\operatorname{arctg} x\,dx = \frac{1}{2}\int 2\frac{1}{1+x^2}\operatorname{arctg} x\,dx = \frac{1}{2}(\operatorname{arctg} x)^2 + C$

- $\int \sec^2 x\sqrt{\operatorname{tg} x}\,dx = \int \frac{1}{\cos^2 x}(\operatorname{tg} x)^{1/2}\,dx = \frac{(\operatorname{tg} x)^{1/2+1}}{\frac{1}{2}+1} = \frac{2}{3}(\operatorname{tg} x)^{3/2} + C$

- $\int \frac{dx}{(3x+1)^4} = \int (3x+1)^{-4}\,dx = \frac{1}{3}\int 3(3x+1)^{-4}\,dx = \frac{1}{3}\frac{(3x+1)^{-4+1}}{-4+1} =$

  $= -\frac{1}{9}(3x+1)^{-3} + C$

- $\int \cos x\cdot \operatorname{sen}^3 x\,dx = \frac{1}{4}\int 4\cos x\,\operatorname{sen}^3 x\,dx = \frac{1}{4}\operatorname{sen}^4 x + C$

- $\int \frac{(x+3)}{(x^2+6x)^{1/3}}\,dx = \int (x+3)(x^2+6x)^{-1/3}\,dx = \frac{1}{2}\int 2(2x+6)(x^2+6x)^{-1/3}\,dx =$

  $= \frac{1}{2}\int (2x+6)(x^2+6x)^{-1/3}\,dx = \frac{1}{2}\frac{(x^2+6x)^{-\frac{1}{3}+1}}{-\frac{1}{3}+1} = \frac{3}{4}(x^2+6x)^{2/3} + C$

- $\int \sqrt{x^2-2x^4}\,dx = \int \sqrt{x^2(1-2x^2)}\,dx = \int x\sqrt{1-2x^2} = \int x(1-2x^2)^{1/2}\,dx =$

  $= -\frac{1}{4}\int -4x(1-2x^2)^{1/2}\,dx = -\frac{1}{4}\frac{(1-2x^2)^{1/2+1}}{\frac{1}{2}+1} = -\frac{1}{6}(1-2x^2)^{3/2} + C$

Integrales inmediatas de tipo exponencial

- $\int x^2 \cdot 7^{x^3+5}\,dx = \frac{1}{3}\int 3x^2 \cdot 7^{x^3+5}\,dx = \frac{1}{3\ln 7}\,7^{x^3+5} + C$

- $\int \frac{5^{\sqrt{x}}}{\sqrt{x}}\,dx = \int \frac{1}{\sqrt{x}}\,5^{\sqrt{x}}\,dx = 2\int \frac{1}{2\sqrt{x}}\,5^{\sqrt{x}}\,dx = \frac{2}{\ln 5}\int \frac{1}{2\sqrt{x}}\,5^{\sqrt{x}}\ln 5\,dx =$

  $= \frac{2}{\ln 5}\,5^{\sqrt{x}} + C$

- $\int \frac{e^{\arcsin x}}{\sqrt{1-x^2}}\,dx = \int \frac{1}{\sqrt{1-x^2}}\,e^{\arcsin x}\,dx = e^{\arcsin x} + C$

- $\int \cos 3x \cdot e^{\operatorname{sen} 3x}\,dx = \frac{1}{3}\int 3\cos 3x \cdot e^{\operatorname{sen} 3x}\,dx = \frac{1}{3}\,e^{\operatorname{sen} 3x} + C$

- $\int \frac{6^{\ln x}}{x}\,dx = \int \frac{1}{x}\,6^{\ln x}\,dx = \frac{1}{\ln 6}\int \frac{1}{x}\,6^{\ln x}\ln 6\,dx = \frac{1}{\ln 6}\,6^{\ln x} + C$

Integrales inmediatas de tipo logarítmico

- $\int \frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x}\,dx = -\int -\frac{\operatorname{sen} x - \cos x}{\operatorname{sen} x + \cos x}\,dx = -\int \frac{\cos x - \operatorname{sen} x}{\operatorname{sen} x + \cos x}\,dx =$

  $= -\ln|\operatorname{sen} x + \cos x| + C$

- $\int \frac{x}{1+x^2}\,dx = \frac{1}{2}\int \frac{2x}{1+x^2}\,dx = \frac{1}{2}\ln|x^2+1| + C$

- $\int \frac{27x^2+30x+3}{3x^3+5x^2+x-1}\,dx = \int \frac{3(9x^2+10x+1)}{3x^3+5x^2+x-1}\,dx = 3\int \frac{9x^2+10x+1}{3x^3+5x^2+x-1}\,dx =$

  $= 3\ln|3x^3+5x^2+x-1| + C$

- $\int \frac{1}{\operatorname{tg} x}\,dx = \int \frac{1}{\frac{\operatorname{sen} x}{\cos x}}\,dx = \int \frac{\cos x}{\operatorname{sen} x}\,dx = \ln|\operatorname{sen} x| + C$

- $\int \frac{7^{2x}}{7^{2x}+5}\,dx = \frac{1}{2}\int \frac{2\cdot 7^{2x}}{7^{2x}+5}\,dx = \frac{1}{2}\,\frac{1}{\ln 7}\int \frac{2\cdot 7^{2x}\cdot\ln 7}{7^{2x}+5}\,dx =$

  $= \frac{1}{2\ln 7}\ln|7^{2x}+5| + C$

- $\displaystyle \int \frac{1}{x\ln x}\,dx = \int \frac{\frac{1}{x}}{\ln x}\,dx = \ln|\ln x| + C$

- $\displaystyle \int \frac{1}{\cos^2 x\cdot \operatorname{tg} x}\,dx = \int \frac{\frac{1}{\cos^2 x}}{\operatorname{tg} x}\,dx = \ln|\operatorname{tg} x| + C$

- $\displaystyle \int \frac{1}{\sqrt{x}\,(1-\sqrt{x})}\,dx = \int \frac{\frac{1}{\sqrt{x}}}{1-\sqrt{x}}\,dx = -2\int \frac{\frac{1}{-2\sqrt{x}}}{1-\sqrt{x}}\,dx = -2\ln|1-\sqrt{x}| + C$

Integración por partes

- $\displaystyle \int \operatorname{arctg} x\,dx = \left[\begin{array}{l} u=\operatorname{arctg} x \longrightarrow du=\frac{1}{1+x^2}\,dx \\ dv = dx \longrightarrow v = x \end{array}\right] = x\cdot\operatorname{arctg} x -$

$\displaystyle -\int x\,\frac{1}{1+x^2}\,dx = x\operatorname{arctg} x - \frac{1}{2}\int \frac{2x}{1+x^2}\,dx = x\operatorname{arctg} x - \frac{1}{2}\ln|1+x^2| + C$

- $\displaystyle \int \operatorname{arcsen} x\,dx = \left[\begin{array}{l} u=\operatorname{arcsen} x \longrightarrow du=\frac{1}{\sqrt{1-x^2}}\,dx \\ dv = dx \longrightarrow v = x \end{array}\right] =$

$\displaystyle = x\operatorname{arcsen} x - \int \frac{x}{\sqrt{1-x^2}}\,dx = x\operatorname{arcsen} x - \frac{1}{2}\,2\underbrace{\int \frac{2x\,(1-x^2)^{-1/2}}{2}\,dx}_{\text{inmediata de tipo potencial}} =$

$\displaystyle = x\operatorname{arcsen} x - (1-x^2)^{1/2} = x\operatorname{arcsen} x - \sqrt{1-x^2} + C$

- $\displaystyle \int \ln(x+1)\,dx = \left[\begin{array}{l} u=\ln(x+1) \longrightarrow du=\frac{1}{x+1}\,dx \\ dv = dx \longrightarrow v = x \end{array}\right] =$

$\displaystyle = x\ln|x+1| - \int x\,\frac{1}{x+1}\,dx = x\ln|x+1| - x + \ln|x+1| + C$

Calculamos $\displaystyle \int \frac{x}{x+1}\,dx = \int 1\,dx - \int \frac{1}{x+1}\,dx = x - \ln|x+1|$

$$\begin{array}{r|l} x & x+1 \\ \cline{2-2} -x-1 & 1 \\ \hline -1 & \end{array}$$

de donde $\displaystyle \frac{x}{x+1} = 1 - \frac{1}{x+1}$

- $\displaystyle \int x\operatorname{arctg} x\,dx = \left[\begin{array}{l} u=\operatorname{arctg} x \longrightarrow du=\frac{1}{1+x^2}\,dx \\ dv = x\,dx \longrightarrow v = \frac{x^2}{2} \end{array}\right] =$

$\displaystyle = \frac{x^2}{2}\operatorname{arctg} x - \int \frac{x^2}{2}\,\frac{1}{1+x^2}\,dx = \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\int \frac{x^2}{1+x^2}\,dx \underset{1}{=}$

Calculamos $\displaystyle\int \frac{x^2}{1+x^2}\,dx = \int 1\,dx - \int \frac{1}{1+x^2}\,dx = x - \operatorname{arctg} x$

$$\begin{array}{l|l} x^2 & x^2+1 \\ \cline{2-2} -x^2-1 & 1 \\ \hline -1 & \end{array} \qquad \text{de donde } \frac{x^2}{1+x^2} = 1 - \frac{1}{1+x^2}$$

$$\underset{(1)}{=} \frac{x^2}{2}\operatorname{arctg} x - \frac{1}{2}\left(x - \operatorname{arctg} x\right) + C$$

- $$\int x \ln(x+1)\,dx = \left[\begin{array}{l} u = \ln(x+1) \longrightarrow du = \frac{1}{x+1}\,dx \\ dv = x\,dx \longrightarrow v = \frac{x^2}{2} \end{array}\right] =$$

$$= \frac{x^2}{2}\ln|x+1| - \int \frac{x^2}{2}\,\frac{1}{x+1}\,dx = \frac{x^2}{2}\ln(x+1) - \frac{1}{2}\int \frac{x^2}{x+1}\,dx \underset{(1)}{=}$$

Calculamos $\displaystyle\int \frac{x^2}{1+x}\,dx = \int (x-1)\,dx + \int \frac{1}{x+1}\,dx = \frac{x^2}{2} - x + \ln|x+1|$

$$\begin{array}{l|l} x^2 & x+1 \\ \cline{2-2} -x^2-x & x-1 \\ \hline -x & \\ x+1 & \\ \hline 1 & \end{array} \qquad \text{de donde } \frac{x^2}{x+1} = x - 1 + \frac{1}{x+1}$$

$$\underset{(1)}{=} \frac{x^2}{2}\ln|x+1| - \frac{1}{2}\left(\frac{x^2}{2} - x + \ln|x+1|\right) + C$$

- $$\int (3x^2+2x-7)\cos x\,dx = \left[\begin{array}{l} u = 3x^2+2x-7 \longrightarrow du = (6x+2)\,dx \\ dv = \cos x\,dx \longrightarrow v = \operatorname{sen} x \end{array}\right] =$$

$$= (3x^2+2x-7)\operatorname{sen} x - \underbrace{\int (6x+2)\operatorname{sen} x\,dx}_{\text{por partes}} \underset{(1)}{=}$$

Calculamos $\displaystyle\int (6x+2)\operatorname{sen} x\,dx = \left[\begin{array}{l} u = 6x+2 \longrightarrow du = 6\,dx \\ dv = \operatorname{sen} x\,dx \longrightarrow v = -\cos x \end{array}\right] =$

$$= -(6x+2)\cos x + 6\int \cos x\,dx = -(6x+2)\cos x + 6\operatorname{sen} x$$

$$\underset{(1)}{=} (3x^2+2x-7)\operatorname{sen} x + (6x+2)\cos x - 6\operatorname{sen} x + C$$

- $$\int (5x^2-3)\,4^{3x+1}\,dx = \left[\begin{array}{l} u = 5x^2-3 \longrightarrow du = 10x\,dx \\ dv = 4^{3x+1}\,dx \longrightarrow v = \underbrace{\int 4^{3x+1}\,dx}_{\substack{\text{inmediata de} \\ \text{tipo exponencial}}} = \frac{1}{4\ln 4}\,4^{3x+1} \end{array}\right] =$$

$$= (5x^2-3)\,\frac{1}{3\ln 4}\,4^{3x+1} - \frac{1}{3\ln 4}\underbrace{\int 10x\cdot 4^{3x+1}\,dx}_{\text{por partes}} \underset{(1)}{=}$$

Calculamos $\int 10x\cdot 4^{3x+1}\,dx = \left[\begin{array}{l} u=10x \longrightarrow du = 10\,dx \\ dv = 4^{3x+1}\,dx \rightarrow v = \frac{1}{3\ln 4}4^{3x+1}\end{array}\right] =$

$$= 10x\,\frac{1}{3\ln 4}4^{3x+1} - \frac{1}{3\ln 4}10\int 4^{3x+1}\,dx =$$

$$= 10x\,\frac{1}{3\ln 4}4^{3x+1} - \frac{1}{3\ln 4}10\,\frac{1}{3\ln 4}4^{3x+1}$$

$$\underset{(1)}{=} (5x^2-3)\,\frac{1}{3\ln 4}4^{3x+1} - \frac{1}{3\ln 4}\left(10x\,\frac{1}{3\ln 4}4^{3x+1} - \frac{1}{3\ln 4}10\,\frac{4^{3x+1}}{3\ln 4}\right) + C$$

- $\int \frac{x}{\operatorname{sen}^2 x}\,dx = \left[\begin{array}{l} u = x \longrightarrow du = dx \\ dv = \frac{1}{\operatorname{sen}^2 x}\,dx \longrightarrow v = \int \frac{1}{\operatorname{sen}^2 x}\,dx = -\operatorname{cotg} x\end{array}\right] =$

$$= -x\operatorname{cotg} x + \int \operatorname{cotg} x\,dx = -x\operatorname{cotg} x + \int \frac{\cos x}{\operatorname{sen} x}\,dx =$$

$$= -x\operatorname{cotg} x + \ln|\operatorname{sen} x| + C$$

- $\int \sec^4 x\,dx = \int \frac{1}{\cos^4 x}\,dx = \int \frac{1}{\cos^2 x\cdot\cos^2 x}\,dx = \int \frac{1}{\cos^2 x}\cdot\frac{1}{\cos^2 x}\,dx =$

$$= \left[\begin{array}{l} u = \frac{1}{\cos^2 x} \longrightarrow du = \frac{2\operatorname{sen} x}{\cos^3 x}\,dx \\ dv = \frac{1}{\cos^2 x}\,dx \longrightarrow v = \operatorname{tg} x = \frac{\operatorname{sen} x}{\cos x}\end{array}\right] = \frac{\operatorname{sen} x}{\cos^3 x} - 2\int \frac{\operatorname{sen}^2 x}{\cos^4 x}\,dx =$$

$$= \frac{\operatorname{sen} x}{\cos^3 x} - 2\int \frac{1-\cos^2 x}{\cos^4 x}\,dx = \frac{\operatorname{sen} x}{\cos^3 x} - 2\int \frac{1}{\cos^4 x}\,dx + 2\int \frac{1}{\cos^2 x}\,dx =$$

$$= \frac{\operatorname{sen} x}{\cos^3 x} - 2\underbrace{\int \frac{1}{\cos^4 x}\,dx}_{\text{Es la integral que estamos calculando}} + 2\operatorname{tg} x \Rightarrow \left(\text{pasando } -2\int \frac{1}{\cos^4 x}\,dx \text{ a la izqda.}\right)$$

$$3\int \frac{1}{\cos^4 x}\,dx = \frac{\operatorname{sen} x}{\cos^3 x} + 2\operatorname{tg} x \Rightarrow \int \frac{1}{\cos^4 x}\,dx = \frac{1}{3}\left(\frac{\operatorname{sen} x}{\cos^3 x} + 2\operatorname{tg} x\right) + C$$

- $\int \frac{\ln x}{\sqrt{x}}\,dx = \left[\begin{array}{l} u = \ln x \longrightarrow du = \frac{1}{x}\,dx \\ dv = \frac{1}{\sqrt{x}}\,dx \longrightarrow v = \int \frac{1}{\sqrt{x}}\,dx = 2\sqrt{x}\end{array}\right] =$

$$= 2\sqrt{x}\ln x - 2\int \frac{\sqrt{x}}{x}\,dx = 2\sqrt{x}\ln x - 4\sqrt{x} + C$$

Integración por cambio de variable

- $$\int \frac{3x}{\sqrt{1+7x^2}}dx = \left[\begin{matrix} t=\sqrt{1+7x^2} \rightarrow t^2 = 1+7x^2 \\ dx = \frac{t}{7x}dt \end{matrix}\right] = \int \frac{3x}{\sqrt{t^2}} \frac{t}{7x} dt =$$

$$= \frac{3}{7}\int dt = \frac{3t}{7} = \frac{3}{7}\sqrt{1+7x^2} + C$$

También se puede resolver como una integral inmediata

- $$\int \frac{5}{x \ln x} dx = \left[\begin{matrix} t = \ln x \rightarrow dt = \frac{1}{x}dx \\ dx = x\,dt \end{matrix}\right] = \int \frac{5}{x \cdot t} x\,dt =$$

$$= 5\int \frac{1}{t}dt = 5\ln|t| = 5\ln|\ln x| + C$$

- $$\int \frac{5}{\sqrt{9-4x^2}}dx = \underbrace{\left[\begin{matrix} 2x = 3\,\text{sen}\,t \rightarrow x = \frac{3}{2}\text{sen}\,t \\ dx = \frac{3}{2}\cos t\,dt \end{matrix}\right]}_{\text{En este caso el cambio de variable nos lo dan}} = \int \frac{5}{\sqrt{9-9\,\text{sen}^2 t}} \frac{3}{2}\cos t\,dt =$$

$$= \frac{15}{2}\int \frac{\cos t}{\sqrt{9}\sqrt{1-\text{sen}^2 t}}dt = \frac{5}{2}\int \frac{\cos t}{\cos t}dt = \frac{5}{2}\int dt = \frac{5}{2}t =$$

$$\boxed{\text{sen}^2 t + \cos^2 t = 1}$$ Hay que sabérsela

$$= \frac{5}{2}\text{arc sen}\left(\frac{2x}{3}\right) + C$$

ya que si $x = \frac{3}{2}\text{sen}\,t \rightarrow \frac{2x}{3} = \text{sen}\,t \rightarrow \text{arcsen}\left(\frac{2x}{3}\right) = \text{arc sen}(\text{sen}\,t) \rightarrow$

$\rightarrow \text{arcsen}\left(\frac{2x}{3}\right) = t$

- $$\int \sqrt{4-x^2}\,dx = \left[\begin{matrix} x = 2\,\text{sen}\,t \rightarrow dx = 2\cos t\,dt \\ \text{El cambio de variable nos lo dan} \end{matrix}\right] = \int \sqrt{4-4\,\text{sen}^2 t}\cdot 2\cos t\,dt =$$

$$= \int 2\cos t \cdot 2\cos t\,dt = 4\int \cos^2 t\,dt = 4\int \frac{1+\cos 2t}{2}dt =$$

$$\boxed{\cos^2 t = \frac{1+\cos 2t}{2}}$$ Hay que sabérsela

$$= 2\int (1+\cos 2t)\,dt = 2\left(t + \frac{\text{sen}\,2t}{2}\right) = 2t + \text{sen}\,2t + C =$$

$$= 2\,\text{arcsen}\left(\frac{x}{2}\right) + \frac{x\sqrt{4-x^2}}{2} + C$$

ya que si $x = 2\,\text{sen}\,t \rightarrow \text{sen}\,t = \frac{x}{2} \rightarrow t = \text{arc sen}\left(\frac{x}{2}\right)$

y $\underbrace{\boxed{\text{sen}(2t) = 2\,\text{sen}\,t\,\cos t}}_{\text{Hay que sabérsela}} = 2\,\text{sen}\,t\cdot\sqrt{1-\text{sen}^2 t} = x\sqrt{1-\left(\frac{x}{2}\right)^2} = \frac{x\sqrt{4-x^2}}{2}$

- $\int \frac{1}{3+2x}\,dx = \left[t = 3+2x \longrightarrow \begin{array}{l} dt = 2\,dx \\ dx = \frac{1}{2}\,dt \end{array} \right] = \int \frac{1}{t}\,\frac{1}{2}\,dt =$

$= \frac{1}{2}\int \frac{1}{t}\,dt = \frac{1}{2}\ln|t| = \frac{1}{2}\ln|3+2x| + C$

También se puede resolver como una integral inmediata

- $\int \frac{x}{2-x^2}\,dx = \left[t = 2-x^2 \rightarrow \begin{array}{l} dt = -2x\,dx \\ dx = \frac{-1}{2x}\,dt \end{array} \right] = \int \frac{1}{t}\cdot\frac{-1}{2x}\,x\,dt =$

$= -\frac{1}{2}\int \frac{1}{t}\,dt = -\frac{1}{2}\ln|t| = -\frac{1}{2}\ln|2-x^2| + C$

- $\int e^{\operatorname{sen} x}\cdot\cos x\,dx = \left[t = \operatorname{sen} x \rightarrow dt = \underbrace{\cos x\,dx}_{\text{es lo que tenemos en la integral}} \right] = \int e^t\,dt =$

$= e^t = e^{\operatorname{sen} x} + C$

- $\int \frac{x}{1+x^4}\,dx = \left[t = x^2 \longrightarrow \begin{array}{l} dt = 2x\,dx \\ \frac{dt}{2} = x\,dx \end{array} \right] = \int \frac{1}{1+t^2}\cdot\frac{1}{2}\,dt =$

$= \frac{1}{2}\int \frac{1}{1+t^2}\,dt = \frac{1}{2}\operatorname{arctg} t = \frac{1}{2}\operatorname{arctg} x^2 + C$

Cambio de variable

① $\int \frac{2^{\ln x}}{x}\,dx = \left[\begin{array}{l} \ln x = t \\ \frac{1}{x}dx = dt \end{array}\right] = \int 2^t dt = \frac{2^t}{\ln 2} + C = \frac{2^{\ln x}}{\ln 2} + C$

② $\int 3x \cos x^2 dx = \left[\begin{array}{l} x^2 = t \\ 2x dx = dt \\ dx = \frac{dt}{2x} \end{array}\right] = \int 3x \cos t \frac{dt}{2x} = \frac{3}{2}\int \cos t\, dt = \frac{3}{2}\operatorname{sen} t =$

$= \frac{3}{2}\operatorname{sen} x^2 + C$

③ $\int \frac{\operatorname{tg} x}{\cos^2 x}dx = \left[\begin{array}{l} \operatorname{tg} x = t \\ \frac{1}{\cos^2 x}dx = dt \end{array}\right] = \int t\,dt = \frac{1}{2}t^2 = \frac{\operatorname{tg}^2 x}{2} + C$

④ $\int \frac{\operatorname{sen} x \cos x}{1+\operatorname{sen}^4 x}dx = \left[\begin{array}{l} \operatorname{sen}^2 x = t \\ 2\operatorname{sen} x \cos x\, dx = dt \end{array}\right] = \frac{1}{2}\int \frac{dt}{1+t^2} = \frac{1}{2}\operatorname{arctg} t =$

$= \frac{1}{2}\operatorname{arctg}(\operatorname{sen}^2 x) + C$

⑤ $\int \frac{\cos\sqrt{2x}}{\sqrt{2x}}dx = \left[\begin{array}{l} \sqrt{2x} = t \\ \frac{1}{\sqrt{2x}}dx = dt \end{array}\right] = -\frac{1}{2}\int \cos t\, dt = \operatorname{sen} t = \operatorname{sen}\sqrt{2x} + C$

Por partes

⑥ $\int x\cos x\,dx = \left[\begin{array}{l} u = x \longrightarrow du = dx \\ dv = \cos x\,dx \longrightarrow v = \operatorname{sen} x \end{array}\right] = x\operatorname{sen} x - \int \operatorname{sen} x\,dx = x\operatorname{sen} x + \cos x + C$

⑦ $\int x^2 e^x dx = \left[\begin{array}{l} u = x^2 \rightarrow du = 2x\,dx \\ dv = e^x dx \rightarrow v = e^x \end{array}\right] = x^2 e^x - \int 2x e^x dx \underset{①}{=}$

Calculamos $\int x e^x dx$ por partes:

$\int x e^x dx = \left[\begin{array}{l} u = x \rightarrow du = dx \\ dv = e^x dx \rightarrow v = e^x \end{array}\right] = xe^x - \int e^x dx = xe^x - e^x$

$\underset{①}{=} x^2e^x - 2(xe^x - e^x) = e^x(x^2 - 2x + 2) + C$

⑧ $\int \cos^2 x\,dx = \int \cos x \cos x\,dx = \left[\begin{array}{l} u = \cos x \longrightarrow du = -\operatorname{sen} x\,dx \\ dv = \cos x\,dx \longrightarrow v = \operatorname{sen} x \end{array}\right] =$

$= \cos x \operatorname{sen} x - \int -\operatorname{sen}^2 dx \underset{\operatorname{sen}^2 x + \cos^2 x = 1}{=} \operatorname{sen} x\cos x + \int (1-\cos^2 x)dx = \operatorname{sen} x \cos x + x - \int \cos^2 x\,dx$

$\Longrightarrow 2\int \cos^2 x\,dx = \operatorname{sen} x\cos x + x \Longrightarrow \int \cos^2 x\,dx = \frac{1}{2}(\operatorname{sen} x \cos x + x) + C$

(9) $\displaystyle\int x^2 \cos x\, dx = \begin{bmatrix} u = x^2 \to du = 2x\,dx \\ dv = \cos x\, dx \to v = \operatorname{sen} x \end{bmatrix} = x^2 \operatorname{sen} x - \int 2x \operatorname{sen} x\, dx \underset{(1)}{=}$

Calculamos $\int x \operatorname{sen} x\, dx$ por partes

$$\int x \operatorname{sen} x\, dx = \begin{bmatrix} u = x \longrightarrow du = dx \\ dv = \operatorname{sen} x\, dx \to v = -\cos x \end{bmatrix} = -x\cos x + \int \cos x\, dx =$$

$$= -x^2 \cos x + \operatorname{sen} x$$

$$\underset{(1)}{=} x^2 \operatorname{sen} x - 2\left(-x^2 \cos x + \operatorname{sen} x\right) = x^2 \operatorname{sen} x + 2x^2 \cos x - 2\operatorname{sen} x + C$$

(10) $\displaystyle\int \frac{x}{2} \operatorname{sen} 3x\, dx = \begin{bmatrix} \frac{x}{2} = u \to du = \frac{1}{2}\,dx \\ dv = \operatorname{sen} 3x\, dx \to -\frac{1}{3}\cos 3x = v \end{bmatrix} = -\frac{1}{6}x\cos 3x + \frac{1}{6}\int \cos 3x\, dx$

$$= -\frac{1}{6} x \cos 3x + \frac{1}{18} \operatorname{sen} 3x + C$$

(11) $\displaystyle\int x\, 3^x\, dx = \begin{bmatrix} u = x \longrightarrow du = dx \\ dv = 3^x dx \to v = \frac{3^x}{\ln 3} \end{bmatrix} = \frac{x\, 3^x}{\ln 3} - \int \frac{3^x}{\ln 3}\, dx =$

$$= \frac{1}{\ln 3}\left(x\, 3^x\right) - \frac{1}{\ln 3}\int 3^x\, dx = \frac{x\, 3^x}{\ln 3} - \frac{1}{\ln 3}\,\frac{3^x}{\ln 3} = \frac{3^x}{\ln 3}\left(x - \frac{1}{\ln 3}\right) + C$$

Integración de funciones racionales

(I) Con solo raíces simples en el denominador

(12) $\displaystyle\int \frac{4x^2+8x+6}{x^3+2x^2-x-2}\, dx$

$$x^3+2x^2-x-2 = (x-1)(x+1)(x+2)$$

$$\begin{array}{r|rrrr}
 & 1 & 2 & -1 & -2 \\
1 & & 1 & 3 & 2 \\
\hline
 & 1 & 3 & 2 & \boxed{0} \\
-1 & & -1 & -2 & \\
\hline
 & 1 & 2 & \boxed{0} & \\
-2 & & -2 & & \\
\hline
 & 1 & \boxed{0} & &
\end{array}$$

$$\frac{4x^2+8x+6}{x^3+2x^2-x-2} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{x+2} =$$

$$= \frac{A(x+1)(x+2) + B(x-1)(x+2) + C(x-1)(x+1)}{(x-1)(x+1)(x+2)}$$

$$4x^2+8x+6 = A(x+1)(x+2) + B(x-1)(x+2) + C(x-1)(x+1) =$$

$$= x^2(A+B+C) + x(3A+B) + (2A-2B-C) \Rightarrow$$

$$\Rightarrow \begin{cases} 4 = A+B+C \\ 8 = 3A+B \\ 6 = 2A-2B-C \end{cases} \longrightarrow (A, B, C) = (3, -1, 2)$$

También se pueden calcular igualando los numeradores y sustituyendo el valor de las raíces del denominador.

$$\int \frac{4x^2+8x+6}{x^3+2x^2-x-2}\,dx = \int \frac{3}{x-1}\,dx + \int \frac{-1}{x+1}\,dx + \int \frac{2}{x+2}\,dx =$$

$$= 3\ln|x-1| - \ln|x+1| + 2\ln|x+2| + C$$

(13) $\displaystyle\int \frac{x+3}{x^2+3x+2}\,dx$

$x^2+3x+2 = (x+1)(x+2)$

$$\begin{array}{r|rrr} & 1 & 3 & 2 \\ -1 & & -1 & -2 \\ \hline & 1 & 2 & \boxed{0} \end{array}$$

$$\frac{x+3}{x^2+3x+2} = \frac{A}{x+1} + \frac{B}{x+2} = \frac{A(x+2)+B(x+1)}{(x+1)(x+2)}$$

$$\left.\begin{array}{l} x=-1 \longrightarrow 2 = A \\ x=-2 \longrightarrow 1 = -B \end{array}\right\} \frac{x+3}{x^2+3x+2} = \frac{2}{x+1} - \frac{1}{x+2}$$

$$\int \frac{x+3}{x^2+3x+2}\,dx = \int \frac{2}{x+1}\,dx - \int \frac{1}{x+2}\,dx = 2\ln|x+1| - \ln|x+2| + C$$

II Con raíces múltiples en el denominador

(14) $\displaystyle\int \frac{2x-1}{x^3-3x^2+3x-1}\,dx$

$x^3-3x^2+3x-1 = (x-1)^3$

$$\begin{array}{r|rrrr} & 1 & -3 & 3 & -1 \\ 1 & & 1 & -2 & 1 \\ \hline & 1 & -2 & 1 & \boxed{0} \\ 1 & & 1 & -1 & \\ \hline & 1 & -1 & \boxed{0} & \end{array}$$

$$\frac{2x-1}{x^3-3x^2+3x-1} = \frac{A}{x-1} + \frac{B}{(x-1)^2} + \frac{C}{(x-1)^3} =$$

$$= \frac{A(x-1)^2+B(x-1)+C}{(x-1)^3}$$

Raíz: $x=1 \Rightarrow C=1$

Valores: $\left\{\begin{array}{l} x=0 \to -1 = A-B \\ x=-1 \to 3 = A+B \end{array}\right\} \to (A,B)=(1,2)$

$$\frac{2x-1}{x^3-3x^2+3x-1} = \frac{1}{x-1} + \frac{2}{(x-1)^2} + \frac{1}{(x-1)^3}$$

$$\int \frac{2x-1}{x^3-3x^2+3x-1}\,dx = \int \frac{1}{x-1}\,dx + 2\int \frac{1}{(x-1)^2}\,dx + \int \frac{1}{(x-1)^3}\,dx =$$

$$= \ln|x-1| - \frac{2}{x-1} - \frac{1}{2(x-1)^2} + C$$

III Con raíces simples y múltiples en el denominador

15 $\displaystyle\int \frac{3x+7}{x^3-x^2-x+1}\,dx$

$$x^3-x^2-x+1 = (x+1)(x-1)^2$$

$$\begin{array}{r|rrrr}
 & 1 & -1 & -1 & 1 \\
1 & & 1 & 0 & -1 \\
\hline
 & 1 & 0 & -1 & \boxed{0} \\
1 & & 1 & 1 & \\
\hline
 & 1 & 1 & \boxed{0} &
\end{array}$$

$$\frac{3x+7}{x^3-x^2-x+1} = \frac{A}{x+1} + \frac{B}{(x-1)} + \frac{C}{(x-1)^2} =$$

$$= \frac{A(x-1)^2 + B(x+1)(x-1) + C(x+1)}{(x+1)(x-1)^2}$$

Raíces: $x=1 \longrightarrow C=5$

$x=-1 \longrightarrow A=1$

Valor: $x=0 \longrightarrow B=-1$

$$\frac{3x+7}{x^3-x^2-x+1} = \frac{1}{x+1} - \frac{1}{x-1} + \frac{5}{(x-1)^2}$$

$$\int \frac{3x+7}{x^3-x^2-x+1}\,dx = \int \frac{1}{x+1}\,dx - \int \frac{1}{x-1}\,dx + 5\int \frac{1}{(x-1)^2}\,dx =$$

$$= \ln|x+1| - \ln|x-1| - \frac{5}{x-1} + C$$