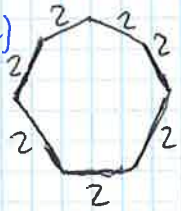
UNIDAD 9.2: LONGITUDES Y ÁREAS

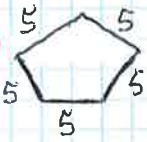
- Perímetro de un polígono → suma de las longitudes de todos sus lados.

① a) 

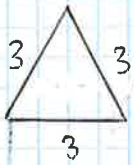
Su perímetro mide 9 cm.
$p = 1'5 + 1'5 + 2 + 2 + 1 + 1 = 9$ cm

b) 

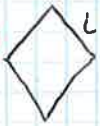
Su perímetro mide
$p = 2 \cdot 7 = 14$ cm.

② a) 

Su perímetro mide
$p = 5 \cdot 5 = 25$ cm.

b) 

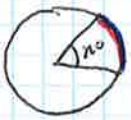
Su perímetro mide
$p = 3 \cdot 3 = 9$ cm.

③ 
$L = \frac{40}{4} = 10$
$p = 40$ cm

$P_{\square} = 4 \cdot 20 = 80$ cm
$2L = 20$

- Longitud de la circunferencia = $2\pi r$

- Longitud de un arco (de circunferencia) = $L_{ARCO} = \frac{2\pi r \cdot n°}{360°}$

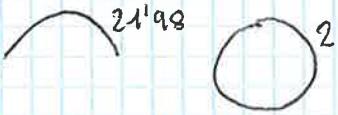


Longitud	$2\pi r$	x
Nº de grados	360	n°

④ a) $r = 4$ cm
$L = 2\pi r = 2 \cdot \pi \cdot 4 = 25'13$ cm.

b) $L_{ARCO\ 30°} = \frac{2 \cdot \pi \cdot r \cdot n°}{360°} = \frac{2 \cdot \pi \cdot 4 \cdot 30}{360} = 2'09$ cm.

c) $L_{ARCO\ 120°} = \frac{2\pi r n°}{360°} = \frac{2 \cdot \pi \cdot 4 \cdot 120}{360} = 8'38$ cm.

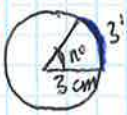
⑤ 

a) ¿r? $L = 2\pi r = 43'96$ Su radio mide 6'99 cm.
$r = \frac{43'96}{2\pi} = 6'99$

b) La circunferencia mide $2 \cdot 21'98 = 43'96$

c) $L_{ARCO\ 40°} = \frac{2\pi r n°}{360°} = \frac{2 \cdot \pi \cdot 6'99 \cdot 40}{360} = 4'88$ cm.
Un arco de 40° mide 4'88 cm.

CALCULAR EL RADIO CON LA LONGITUD
$r = \frac{L}{2\pi} = \frac{43'96}{2\pi} = 6'99$

⑥ 

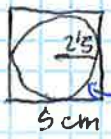
$L_{ARCO} = \frac{2 \cdot \pi \cdot r \cdot n°}{360°} = \frac{2 \cdot \pi \cdot r \cdot n}{360} = \frac{3'14}{1}$

$\frac{2\pi \cdot 3 \cdot n}{360} = \frac{1130'4}{360}$

Número
$2 \cdot \pi \cdot 3 \cdot n = 1130'4$

$n = \frac{1130'4}{2 \cdot \pi \cdot 3} = 59'97$

7)

a) 

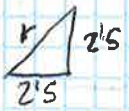
Circunferencia inscrita

$L_{CIRCUNFERENCIA\ INSCRITA} = 2 \cdot \pi \cdot r = 2 \cdot \pi \cdot 2'5 = 15'7$ cm

b) 

circunferencia circunscrita

$L_{CIRCUNFERENCIA\ CIRCUNSCRITA} = 2 \cdot \pi \cdot r = 2 \cdot \pi \cdot 3'54 = 22'54$ cm



T^a de Pitágoras

$r^2 = 2'5^2 + 2'5^2$

$r^2 = 12'5$

$r = \sqrt{12'5} = 3'54$

La diferencia que observo es que:

$L_{CIRCUNFERENCIA\ INSCRITA} < L_{CIRCUNFERENCIA\ CIRCUNSCRITA}$

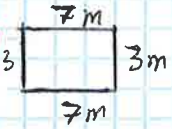
FÓRMULAS ÁREAS

Rectángulo

$A_{RECTÁNGULO} = b \cdot h$

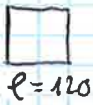
Cuadrado

$A_{CUADRADO} = l^2$

8) 

$A = b \cdot h = 7 \cdot 3 = 21\ m^2$

$P = 7 + 3 + 7 + 3 = 20\ m$

9) 

$A = l^2 = 1200^2 = 1440000\ m^2$.

10) a) 

$P = 120\ cm \Rightarrow l = \frac{120}{4} = 30\ cm$

$A = l^2 = 30^2 = 900\ cm^2$

b) 

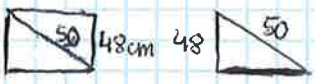
$P = 260\ cm \Rightarrow l = \frac{260}{4} = 65\ cm$

$A = l^2 = 65^2 = 4225\ cm^2$.

11) 

$A = 24'5\ cm^2 \quad h = \frac{24'5}{3'5} = 7$

$A = b \cdot h = 3'5 \cdot 7 = 24'5\ cm^2$

12) 

$50^2 = 48^2 + x^2$

$2500 = 2304 + x^2$

$2500 - 2304 = x^2$

$196 = x^2$

$14 = \sqrt{196} = x$

$A = b \cdot h = 14 \cdot 48 = \underline{672\ cm^2}$

$P = 14 + 48 + 14 + 48 = \underline{124\ cm}$

13) a) 

$10^2 = x^2 + x^2$

$100 = 2x^2$

$\frac{100}{2} = x^2$

$50 = x^2$

$7'07 = \sqrt{50} = x$

$A = l^2 = 7'07^2 = 49'98\ cm^2$

$P = 4 \cdot 7'07 = 28'28\ cm$

b) 

$20^2 = x^2 + x^2$

$400 = 2x^2$

$\frac{400}{2} = x^2$

$200 = x^2$

$14'14 = \sqrt{200} = x$

$A = l^2 = 14'14^2 = 199'94\ cm^2$

$P = 4 \cdot 14'14 = 56'56\ cm$.

c) 

$30^2 = x^2 + x^2$

$900 = 2x^2$

$\frac{900}{2} = x^2$

$450 = x^2$

$21'21 = \sqrt{450} = x$

$A = l^2 = 21'21^2 = 449'86\ cm^2$.

$P = 4 \cdot 21'21 = 84'84\ cm$.

(14)  3000 m

4'5 km = 4500 m

$A = b \cdot h = 4500 \cdot 3000 = 13500000 \ m^2$
$= 1350 \ ha$

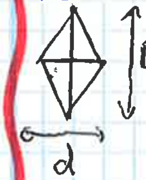
Precio $= 13500000 \cdot 3'60 = 48600000$ €

$1 \ ha = 1 \ hm^2$
Hectárea — Hectómetro

$km^2 \quad hm^2 \quad dam^2 \quad m^2$ ($\cdot 100$ / $: 100$)

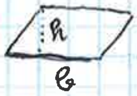
FÓRMULAS ÁREAS

Rombo

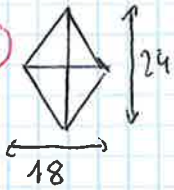


$A_{ROMBO} = \frac{D \cdot d}{2}$

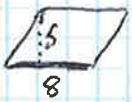
Romboide



$A_{ROMBOIDE} = b \cdot h$

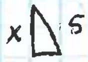
(15) 

$A = \frac{D \cdot d}{2} = \frac{24 \cdot 18}{2} = 216 \ cm^2$

(16) 

$A = b \cdot h = 8 \cdot 5 = 40 \ cm^2$.

(17) 



$6 : 2 = 3$

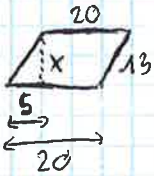
$P = 20$

$20 : 4 = 5 =$ lado

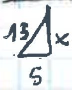
$5^2 = 3^2 + x^2$
$25 = 9 + x^2$
$25 - 9 = x^2$
$16 = x^2$
$\sqrt{16} = x$
$4 = x$

$4 \cdot 2 = 8 \ cm$

$A = \frac{D \cdot d}{2} = \frac{8 \cdot 6}{2} = 24 \ cm^2$

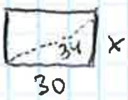
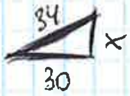
(18) 

$P = 20 + 13 + 20 + 13 = 66 \ cm$.



$13^2 = 5^2 + x^2$
$169 = 25 + x^2$
$169 - 25 = x^2$
$144 = x^2$
$12 = \sqrt{144} = x$

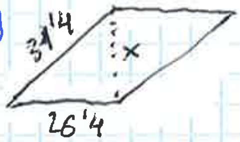
$A = b \cdot h = 20 \cdot 12 = 240 \ cm^2$.

(19) a)  

$34^2 = 30^2 + x^2$
$1156 = 900 + x^2$
$1156 - 900 = x^2$
$256 = x^2$
$16 = \sqrt{256} = x$

$P = 30 + 16 + 30 + 16 = 92 \ cm$.

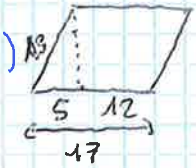
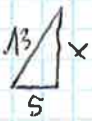
$A = b \cdot h = 30 \cdot 16 = 480 \ cm^2$.

b) 

$P = 31'4 + 26'4 + 31'4 + 26'4 = 115'6 \ cm$.

$A = b \cdot h = 26'4 \cdot 17 = 448'8 \ cm^2$.

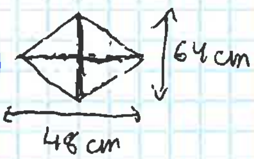
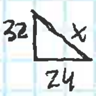
$31'4^2 = 26'4^2 + x^2$
$985'96 = 696'96 + x^2$
$985'96 - 696'96 = x^2$
$289 = x^2$
$17 = \sqrt{289} = x$

c)  

$13^2 = 5^2 + x^2$
$169 = 25 + x^2$
$169 - 25 = x^2$
$144 = x^2$
$12 = \sqrt{144} = x$

$A = b \cdot h = 17 \cdot 12 = 204 \ cm^2$.

$P = 17 + 13 + 17 + 13 = 60 \ cm$.

d)  

$x^2 = 32^2 + 24^2$
$x^2 = 1024 + 576$
$x^2 = 1600$
$x = \sqrt{1600} = 40$

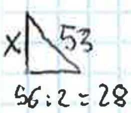
$P = 4 \cdot 40 = 160 \ cm$.

$A = \frac{D \cdot d}{2} = \frac{64 \cdot 48}{2} = 1536 \ cm^2$.

(20) 

$P = 212$

$212 : 4 = 53$

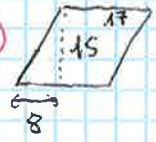


$56 : 2 = 28$

$53^2 = 28^2 + x^2$
$2809 = 784 + x^2$
$2809 - 784 = x^2$
$2025 = x^2$
$45 = \sqrt{2025} = x$

$D = 45 \cdot 2 = 90$

$A = \frac{D \cdot d}{2} = \frac{90 \cdot 56}{2} = 2520 \ cm^2$.

21 

a) 

$x^2 = 15^2 + 8^2$
$x^2 = 225 + 64$
$x^2 = 289$
$x = \sqrt{289} = 17$

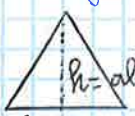
$P = 17 \cdot 4 = 68$ cm.

b) Es un rombo porque sus cuatro lados son iguales y sus ángulos iguales 2 a 2.

c) Usaría la siguiente fórmula: $A = \frac{D \cdot d}{2}$.

FÓRMULAS ÁREAS

Triángulo



$A_{TRIÁNGULO} = \frac{b \cdot h}{2}$

22 a) 

$A_{TRIÁNGULO} = \frac{b \cdot h}{2} = \frac{7 \cdot 4}{2} = 14\ cm^2$.

b) 

$A_{TRIÁNGULO} = \frac{b \cdot h}{2} = \frac{5 \cdot 2}{2} = 5\ cm^2$.

25 

$A = \frac{b \cdot h}{2} = 12 = \frac{4 \cdot h}{2}$

Su altura mide 6 cm.

a)

$\frac{24}{2} = \frac{4 \cdot h}{2}$

$24 = 4 \cdot h$

$6 = \frac{24}{4} = h$

b) 

$A = \frac{b \cdot h}{2} = 15 = \frac{5 \cdot h}{2}$

Su altura mide 6 cm.

$\frac{30}{2} = \frac{5 \cdot h}{2}$

$30 = 5 \cdot h$

$6 = \frac{30}{5} = h$

26 a) $A = \frac{b \cdot h}{2} = 20 = \frac{b \cdot 10}{2}$

$\frac{40}{2} = \frac{b \cdot 10}{2}$

$40 = b \cdot 10$

$4 = \frac{40}{10} = b$

La base mide 4 cm.

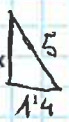
b) $A = \frac{b \cdot h}{2} = 21 = \frac{b \cdot 7}{2}$

$\frac{42}{2} = \frac{b \cdot 7}{2}$

$42 = b \cdot 7$

$6 = \frac{42}{7} = b$

La base mide 6 cm.

28 a) 

$5^2 = 1'4^2 + x^2$
$25 = 1'96 + x^2$
$25 - 1'96 = x^2$
$23'04 = x^2$
$4'8 = \sqrt{23'04} = x$

$A = \frac{b \cdot h}{2} = \frac{1'4 \cdot 4'8}{2} = 6'72\ cm^2$.

b) 

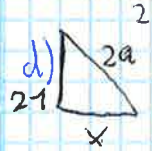
$73^2 = 48^2 + x^2$
$5329 = 2304 + x^2$
$5329 - 2304 = x^2$
$3025 = x^2$
$55 = \sqrt{3025} = x$

$A = \frac{b \cdot h}{2} = \frac{48 \cdot 55}{2} = 1320\ cm^2$.

c) 

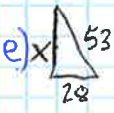
$6'8^2 = 3'2^2 + x^2$
$46'24 = 10'24 + x^2$
$46'24 - 10'24 = x^2$
$36 = x^2$
$6 = \sqrt{36} = x$

$A = \frac{b \cdot h}{2} = \frac{6 \cdot 3'2}{2} = 9'6\ cm^2$.

d) 

$29^2 = 21^2 + x^2$
$841 = 441 + x^2$
$841 - 441 = x^2$
$400 = x^2$
$20 = \sqrt{400} = x$

$A = \frac{b \cdot h}{2} = \frac{20 \cdot 21}{2} = 210\ cm^2$.

e) 

$53^2 = 28^2 + x^2$
$2809 = 784 + x^2$
$2809 - 784 = x^2$
$2025 = x^2$
$45 = \sqrt{2025} = x$

$A = \frac{b \cdot h}{2} = \frac{28 \cdot 45}{2} = 630\ cm^2$.

g) 

$6'5^2 = 2'5^2 + x^2$
$42'25 = 6'25 + x^2$
$42'25 - 6'25 = x^2$
$36 = x^2$
$6 = \sqrt{36} = x$

$A = \frac{b \cdot h}{2} = \frac{2'5 \cdot 6}{2} = 7'5\ cm^2$.

29 a) $A = \frac{b \cdot h}{2} = \frac{6 \cdot 7}{2} = 21\ cm^2$. b) $A = \frac{b \cdot h}{2} = 12'5\ cm^2$.

c) $10^2 = 8^2 + x^2$; $100 = 64 + x^2$; $100 - 64 = x^2$; $36 = x^2$; $6 = \sqrt{36} = x$

$A = \frac{b \cdot h}{2} = \frac{8 \cdot 6}{2} = 24\ cm^2$.

d) $15^2 = 12^2 + x^2$; $225 = 144 + x^2$; $225 - 144 = x^2$; $81 = x^2$; $9 = \sqrt{81} = x$

$A = \frac{b \cdot h}{2} = \frac{12 \cdot 9}{2} = 54\ cm^2$.

FORMULAS ÁREAS

Trapecio

b = base menor

h = altura

B = base mayor

$A_{TRAPECIO} = \frac{(B+b) \cdot h}{2}$

Polígono regular

P = perímetro

a = apotema

L = lado

$A_{POLÍGONO\ REGULAR} = \frac{P \cdot a}{2}$

37 $A = \frac{(B+b) \cdot h}{2} = \frac{(5+4) \cdot 2}{2} = 9\ cm^2$.

38 $A_{TRAPECIO} = \frac{(B+b) \cdot h}{2}$

$\frac{15}{1} = \frac{(6+b) \cdot 3}{2}$

$\frac{30}{2} = \frac{(6+b) \cdot 3}{2}$

$30 = 6 + b \cdot 3$

$30 = 18 + 3b$

$30 - 18 = 3b$

$12 = 3b$

$4 = \frac{12}{3} = b$

La base menor mide 4 cm.

40 a) $10^2 = 6^2 + h^2$; $100 = 36 + h^2$; $100 - 36 = h^2$; $64 = h^2$; $8 = \sqrt{64} = h$.

$A = \frac{(B+b) \cdot h}{2} = \frac{(24+12) \cdot 8}{2} = 144\ cm^2$.

b) $13^2 = 5^2 + h^2$; $169 = 25 + h^2$; $169 - 25 = h^2$; $144 = h^2$; $12 = \sqrt{144} = h$

$A = \frac{(B+b) \cdot h}{2} = \frac{(20+10) \cdot 12}{2} = 180\ cm^2$.

41 Opción 1

$h = 5$

$A = \frac{(B+b) \cdot h}{2} = \frac{(8+5) \cdot 5}{2} = 32'5\ cm^2$.

$x^2 = 5^2 + 3^2$; $x^2 = 25 + 9$; $x^2 = 34$; $x = \sqrt{34} = 5'83$

$P = 5'83 + 8 + 5 + 5 = 24'83\ cm$

Opción 2

$5^2 = 3^2 + h^2$; $25 = 9 + h^2$; $25 - 9 = h^2$; $16 = h^2$; $4 = \sqrt{16} = h$

$h = 4$

$A = \frac{(B+b) \cdot h}{2} = \frac{(8+5) \cdot 4}{2} = 26\ cm^2$.

$P = 5 + 8 + 5 + 4 = 22\ cm$.

44 a) $A = \frac{P \cdot a}{2} = \frac{48 \cdot 7'24}{2} = 173'76\ cm^2$.

$P = 6 \cdot 8 = 48$

b) $P = 5 \cdot 7 = 35$; $A = \frac{P \cdot a}{2} = \frac{35 \cdot 5'19}{2} = 90'82\ cm^2$

c) $L = 3$; $P = 3 \cdot 5 = 15$; $A = \frac{P \cdot a}{2} = \frac{15 \cdot 2'06}{2} = 15'45\ cm^2$

d) $P = 10 \cdot 6 = 60$; $A = \frac{P \cdot a}{2} = \frac{60 \cdot 9'23}{2} = 276'9\ cm^2$.

45 ¿a? $A = \frac{P \cdot a}{2}$

$l = 8\ cm$.

$A = 110\ cm^2$.

$\frac{110}{1} = \frac{40 \cdot a}{2}$

$\frac{220}{2} = \frac{40a}{a}$

$220 = 40a$

$5'5 = \frac{220}{40} = a$

La apotema mide 5'5 cm.

46 a) $\frac{1}{2} A$

b) $\frac{4}{6} A = \frac{2}{3} A$

c)  $\frac{3}{6} A = \frac{1}{2} A$

47 a)  $7'42^2 = 6^2 + x^2$

$55'06 = 36 + x^2$

$55'06 - 36 = x^2$

$19'06 = x^2$

$4'37 = \sqrt{19'06} = x$

$P = 4'37 \cdot 5 = 21'85$ cm.

$A = \frac{P \cdot a}{2} = \frac{21'85 \cdot 6}{2} =$

$131'1$ cm^2.

b)  $9^2 = 5'29^2 + a^2$

$81 = 27'98 + a^2$

$81 - 27'98 = a^2$

$53'02 = a^2$

$7'28 = \sqrt{53'02} = a$

$P = 10'58 \cdot 5 = 52'9$

$A = \frac{P \cdot a}{2} = \frac{52'9 \cdot 7'28}{2} = 192'56$ cm^2.

c)  $5^2 = 4'5^2 + x^2$

$25 = 20'25 + x^2$

$25 - 20'25 = x^2$

$4'75 = x^2$

$2'18 = \sqrt{4'75} = x$

$\ell = 2'18 \cdot 2 = 4'36$

$P = 4'36 \cdot 7 =$

$30'52$ cm.

$A = \frac{P \cdot a}{2} = \frac{30'52 \cdot 4'5}{2} = 68'67$ cm^2.

d)  $9'22^2 = 4^2 + a^2$

$85'01 = 16 + a^2$

$85'01 - 16 = a^2$

$69'01 = a^2$

$8'31 = \sqrt{69'01} = a$

$P = 8 \cdot 7 = 56$ cm.

$A = \frac{P \cdot a}{2} = \frac{56 \cdot 8'31}{2} = 232'68$ cm^2.

e) $\ell = 6'84$

$r = 10$

$a = ?$

$10^2 = 3'42^2 + a^2$

$100 = 11'70 + a^2$

$100 - 11'70 = a^2$

$88'3 = a^2$

$9'40 = \sqrt{88'3} = a$

$P = 6'84 \cdot 9 = 61'56$ cm.

$A = \frac{P \cdot a}{2} = \frac{61'56 \cdot 9'40}{2} = 289'33$ cm^2.

f) $\ell = 4'14$ cm.

$r = 8$ cm.

$a = ?$

$8^2 = 2'07^2 + a^2$

$a = \sqrt{8^2 - 2'07^2} = 7'73$

$P = 4'14 \cdot 12 = 49'68$ cm.

$A = \frac{P \cdot a}{2} = \frac{49'68 \cdot 7'73}{2} = 192'01$ cm^2.

FÓRMULAS ÁREAS

Círculo

$A_{CÍRCULO} = \pi \cdot r^2$

$L_{CIRCUNFERENCIA} = 2 \cdot \pi \cdot r$

Sector circular

$A = \frac{\pi \cdot r^2 \cdot n^\circ}{360^\circ}$

$L = 2 \cdot \pi \cdot r \cdot \frac{n^\circ}{360^\circ}$

50 a) $r = 2'5$ cm.

$A = \pi \cdot r^2 = \pi \cdot 2'5^2 = 19'63$ cm^2.

b) $d = 12$ cm $\rightarrow r = 6$ cm

$A = \pi \cdot 6^2 = 113'10$ cm^2.

c) $L = 62'8$ cm $\rightarrow 2\pi \cdot r = 62'8$ cm.

$r = \frac{62'8}{2\pi} = 9'99$

$A = \pi \cdot r^2 =$

$\pi \cdot 9'99^2 = 313'53$ cm^2.

51 a) $r = 4$ cm, $n^\circ = 20^\circ$ $\Big\} A = \frac{\pi \cdot r^2 \cdot n^\circ}{360^\circ} = \frac{\pi \cdot 4^2 \cdot 20}{360} = 2'79$ cm^2.

b) $r = 7$ cm, $n^\circ = 150^\circ$ $\Big\} A = \frac{\pi \cdot r^2 \cdot n^\circ}{360^\circ} = \frac{\pi \cdot 7^2 \cdot 150}{360} = 64'14$ cm^2.

c) $r = 10$ cm, $n^\circ = 180^\circ$ $\Big\} A = \frac{\pi \cdot r^2 \cdot n^\circ}{360^\circ} = \frac{\pi \cdot 10^2 \cdot 180}{360} = 157'07$ cm^2.

54 $A_{FIGURA} = A_{TRIÁNGULO} - \frac{1}{2} A_{CÍRCULO} =$

a) $= \frac{6 \cdot h}{2} - \frac{1}{2} \cdot \pi \cdot r^2$

$= \frac{6 \cdot 4}{2} - \frac{1}{2} \cdot \pi \cdot 2^2 = 5'72$ cm^2.

$5^2 = 3^2 + h^2$

$25 = 9 + h^2$

$25 - 9 = h^2$

$16 = h^2$

$4 = \sqrt{16} = h$

b) $A_{FIGURA} = A_{RECTÁNGULO} - \frac{1}{2} A_{CÍRCULO}$

$= 6 \cdot h - \frac{1}{2} \cdot \pi \cdot 3^2 =$

$= 6 \cdot 6'4 - \frac{1}{2} \cdot \pi \cdot 3^2 = 24'26$ cm^2.

d) $A_{FIGURA} = A_{PENTÁGONO} - A_{CÍRCULO} =$

$= \frac{P \cdot a}{2} - \pi \cdot r^2 =$ $P = 2 \cdot 5 = 10$

$= \frac{10 \cdot 1'38}{2} - \pi \cdot 1'38^2 = 2'56$ cm^2.